Quiz 6
Chemical Engineering Thermodynamics
February 18, 2021

ULT-Freezers (ultra-low temperature) are needed for storage of COVID-19 mRNA vaccine and other nucleic acids (DNA and RNA) in order to prevent degradation reactions. Consider a **5-ton** rated binary cascade refrigerator to cool COVID-19 mRNA vaccine. **Find the COP for the cascade refrigerator and that for a Carnot cycle. Stage 1 uses R134A refrigerant and Stage 2 uses ethane. The condenser (8) is at 30°C, the inter-stage heat exchanger (6, 4) is at -30°C and the evaporator (2) is at -86°C.** Assume that the heat exchanger has no thermal loss.
Use the closest values from the saturated table for R134A for the saturated values and the pressure-enthalpy chart for the other values;
and **interpolate the values from the saturated table for ethane for the saturated values** and use the pressure-enthalpy chart for ethane for the other values (*use the attached Excel sheet to do the interpolation by inserting the values from the table*). For -86°C (187K) use the saturated pressure from the saturated table interpolation to find the equilibrium tie-line in the chart (1 to 2 in the schematic chart below).
The two compressors have an **efficiency of 0.85**.
1 ton refrigeration = 12,600 kJ/h

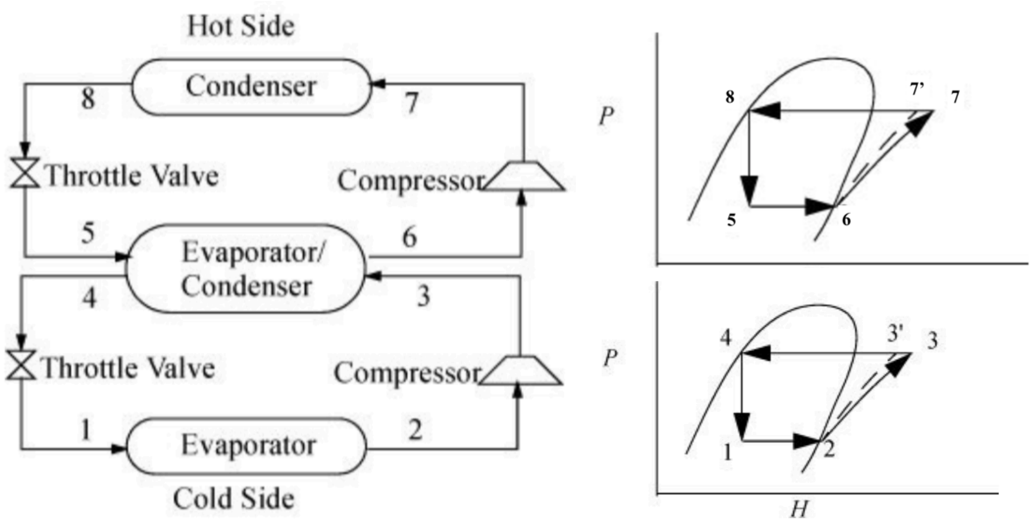


Figure 1. Cascade refrigeration cycle. The refrigerants do not mix in the evaporator/condenser. P-H diagrams for the upper and the lower cycles.

Fill the table values in the process stream table.
Plot the process stream points on the two P-H charts with lines connecting as in Figure 1 above.

PROPERTIES OF SATURATED ETHANE

T	P	volume, m^3/kg		enthalpy, kJ/kg			entropy, kJ/(kg·K)		
K	MPa	v_f	v_g	h_f	h_{fg}	h_g	s_f	s_{fg}	s_g
150	0.009591	0.001693	4.299	0.0	539.18	539.18	0.0	3.5945	3.5945
160	0.02153	0.001735	2.031	27.17	523.99	551.16	0.1752	3.2750	3.4502
170	0.04329	0.001779	1.064	53.61	509.29	562.90	0.3353	2.9959	3.3312
180	0.07968	0.001823	0.6053	79.62	494.67	574.29	0.4836	2.7482	3.2318
184.32	0.101325	0.001843	0.4845	90.80	488.28	579.08	0.5448	2.6491	3.1939
190	0.1364	0.001869	0.3676	105.51	479.72	585.23	0.6230	2.5249	3.1479
200	0.2200	0.001918	0.2355	131.50	464.11	595.61	0.7555	2.3206	3.0761
210	0.3376	0.001969	0.1575	157.69	447.64	605.33	0.8822	2.1316	3.0138
220	0.4968	0.002023	0.1091	184.13	430.15	614.28	1.0037	1.9553	2.9590
230	0.7057	0.002082	0.07768	210.90	411.44	622.34	1.1208	1.7889	2.9097
240	0.9730	0.002147	0.05655	238.21	391.10	629.31	1.2346	1.6296	2.8642
250	1.308	0.002221	0.04184	266.47	368.49	634.96	1.3470	1.4739	2.8209
260	1.720	0.002308	0.03130	296.21	342.70	638.91	1.4599	1.3181	2.7780
270	2.221	0.002415	0.02352	327.97	312.59	640.56	1.5753	1.1577	2.7330
280	2.822	0.002555	0.01759	362.34	276.44	638.78	1.6948	0.9873	2.6821
290	3.541	0.002759	0.01286	400.71	230.25	630.96	1.8228	0.7939	2.6167
300	4.409	0.003142	0.008602	450.52	156.97	607.49	1.9828	0.5232	2.5060
305.88	5.010	0.004596	0.004596	532.03	0.0	532.03	2.2441	0.0	2.2441

Stream	P, Mpa	T, °C	η_e	State	H, kJ/kg	S, kJ/(kgK)	q	ΔQ or W_s, kJ/kg	m', kg/h (kg/s)	$\underline{\Delta Q}$ or $\underline{W}_s$, kJ/h
ETHANE										
1	0.118	-86	-	V/L	246	1.38	0.3	0	188 (0.0522)	0
2	0.118	-86	-	SV	582	3.17	1	336	188 (0.0522)	63,200
3'	1.07	17	1	V	726	3.17	1	144	188 (0.0522)	27,100
3	1.07	32	0.85	V	751	3.3	1	169	188 (0.0522)	31,800
4	1.07	-30	-	SL	246	1.27	0	-505	188 (0.0522)	-94,900
R134a										
5	0.0878	-30	-	V/L	243	1.17	0.365	0	688 (0.191)	0
6	0.0878	-30	-	SV	381	1.75	1	138	688 (0.191)	94,900
7'	0.789	40	1	V	422	1.75	1	41	688 (0.191)	28,200
7	0.789	43	0.85	V	429	1.77	1	48.2	688 (0.191)	33,200
8	0.789	30	-	SL	243	1.15	0	-186	688 (0.191)	-128,000
Net COP =	0.972	Carnot COP =	1.61							

$$\text{Carnot COP} = \frac{273K + (-86°C)}{30° - (-86°C)} = 1.61$$

$$H_5 = 0.365\left(381 \tfrac{kJ}{kg}\right) + (1-0.365)\left(162 \tfrac{kJ}{kg}\right) = 484\ kJ/kg$$

$$S_5 = 0.365\left(0.853 \tfrac{kJ}{kg}\right) + (1-0.365)\left(1.715 \tfrac{kJ}{kg}\right) = 1.42\ kJ/kg\,K$$

$$\dot{Q}_{C_{Ethane}} = H_2 - H_1 = 336 \frac{kJ}{kg}$$

$$5\ \text{ton} \cdot 12{,}600\ kJ/h = 63{,}000 \frac{kJ}{h}$$

$$\dot{m}_{ethane} = \frac{63{,}000\ kJ/h}{336\ kJ/kg} = 188\ kg/h$$

$$\left(188\ kg/h\right)\left(\frac{1\ hr}{3600\ s}\right) = 0.0522\ kg/s$$

$$\dot{m}_{ethane} Q_{H_{Ethane}} = -\dot{m}_{R134a} Q_{C_{R134a}}$$

$$Q_{H_{Ethane}} = H_4 - H_3 = -505\ kJ/kg$$

$$Q_{C_{134a}} = H_6 - H_5 = 138\ kJ/kg$$

$$\dot{m}_{134a} = \frac{-(188\ kJ/h)(-505\ kJ/kg)}{138\ kJ/kg}$$

$$\dot{m}_{134a} = 688\ kg/h \qquad 688 \frac{kg}{h}\left(\frac{1h}{3600s}\right) = 0.191 \frac{kg}{s}$$

$$COP = \frac{\dot{Q}_{C\,2\text{-}1}}{H_{7\text{-}6} + H_{3\text{-}2}} = \frac{63{,}200\ kJ/h}{33{,}200 \frac{kJ}{h} + 31{,}800 \frac{kJ}{h}} = 0.972$$

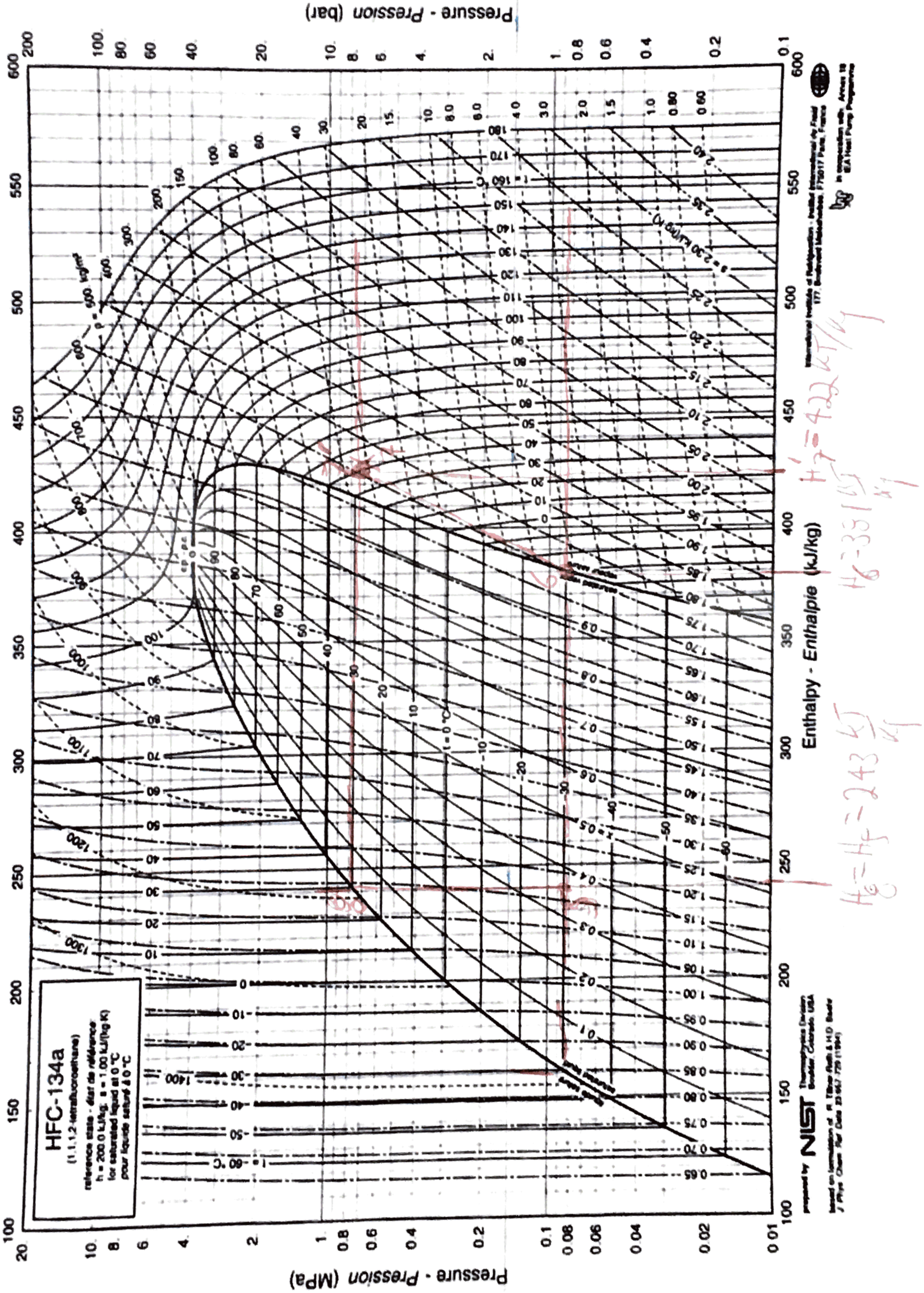
-30° -86°C (187K) -30°C (243K) 30°C (303K) 30°

T	P	ρ^L	ρ^V	H^L	H^V	S^L	S^V
K	MPa	kg/m^3	kg/m^3	kJ/kg	kJ/kg	kJ/kg-K	kJ/kg-K
240	0.07248	1397.7	3.8367	156.78	378.33	0.8320	1.7552
244	0.08784	1385.8	4.5965	161.87	380.85	0.8530	1.7505
248	0.10568	1373.8	5.4707	166.99	383.35	0.8738	1.7462
252	0.12627	1361.7	6.4715	172.14	385.84	0.8943	1.7423
256	0.14989	1349.5	7.6117	177.33	388.31	0.9147	1.7388
260	0.17684	1337.0	8.9051	182.55	390.75	0.9348	1.7356
264	0.20742	1324.4	10.3660	187.81	393.17	0.9548	1.7327
268	0.24197	1311.6	12.0110	193.11	395.56	0.9747	1.7301
272	0.28080	1298.5	13.8570	198.45	397.93	0.9943	1.7277
276	0.32426	1285.3	15.9230	203.84	400.25	1.0139	1.7255
280	0.37271	1271.7	18.2270	209.26	402.54	1.0332	1.7235
284	0.42651	1258.0	20.7940	214.74	404.79	1.0525	1.7217
288	0.48603	1243.9	23.6450	220.27	406.99	1.0717	1.7200
292	0.55165	1229.5	26.8080	225.85	409.14	1.0907	1.7184
296	0.62378	1214.7	30.3130	231.49	411.23	1.1097	1.7169
300	0.70282	1199.6	34.1920	237.18	413.26	1.1286	1.7155
304	0.78918	1184.1	38.4830	242.95	415.22	1.1475	1.7142
308	0.88330	1168.1	43.2280	248.78	417.11	1.1663	1.7128
312	0.98560	1151.5	48.4750	254.69	418.92	1.1850	1.7114
316	1.09650	1134.5	54.2820	260.68	420.63	1.2038	1.7100
320	1.21660	1116.7	60.7140	266.76	422.25	1.2226	1.7085
324	1.34620	1098.3	67.8510	272.94	423.74	1.2414	1.7068
328	1.48600	1079.0	75.7890	279.23	425.10	1.2603	1.7050
332	1.63640	1058.8	84.6440	285.63	426.31	1.2793	1.7030
336	1.79810	1037.5	94.5630	292.18	427.34	1.2984	1.7007
340	1.97150	1015.0	105.7300	298.88	428.17	1.3177	1.6980

Abstracted from R. Tillner-Roth; H. D. Baehr, 1994. *J. Phys. Chem. Ref. Data*, 23:657.

E.12 PRESSURE-ENTHALPY DIAGRAM FOR R134A (1,1,1,2-TETRAFLUOROETHANE)

(Source: NIST, Thermophysics Division, Boulder, CO, USA, used with permission.



PROPERTIES OF SATURATED ETHANE

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190	0.1364	0.001869	0.3676	105.51	479.72	585.23	0.6230	2.5249	3.1479
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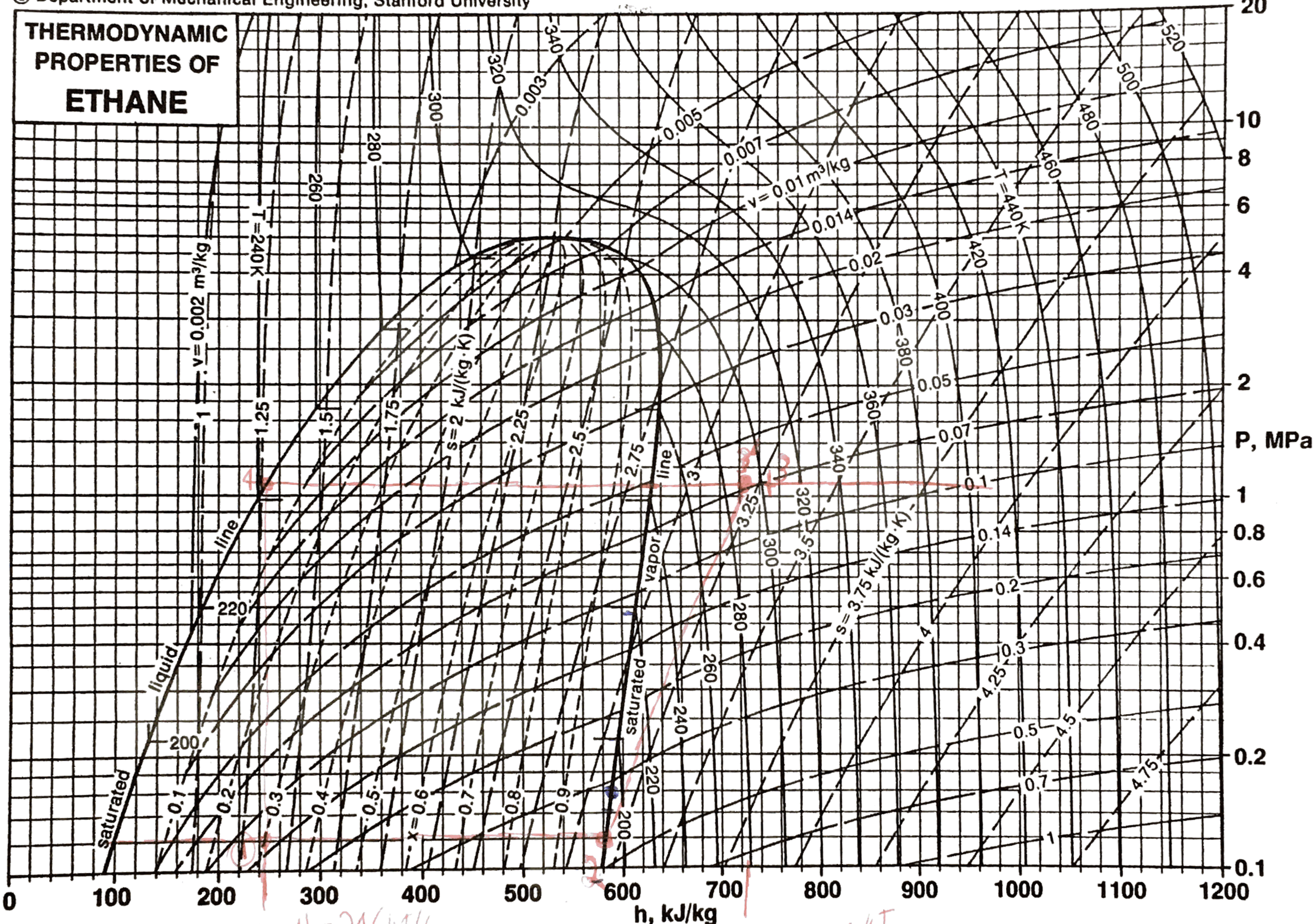
305.88	5.010	0.004596	0.004596	532.03	0.0	532.03	2.2441	0.0	2.2441

PROPERTIES OF GASEOUS ETHANE

P,MPa (T_{sat},K)		T,K: sat	300	340	380	420	460	500	540	580
0.070	v,m³/kg	0.6824	1.180	1.340	1.499	1.658	1.816	1.975	2.134	2.292
(177.8)	h,kJ/kg	571.78	764.36	838.62	919.15	1006.11	1099.55	1199.46	1305.74	1418.27
	s,kJ/(kg·K)	3.2525	4.0686	4.3007	4.5245	4.7419	4.9543	5.1625	5.3669	5.5679
0.101325	v,m³/kg	0.4845	0.8132	0.9240	1.034	1.144	1.254	1.364	1.474	1.583
(184.3)	h,kJ/kg	579.08	763.73	838.14	918.77	1005.80	1099.29	1199.24	1305.56	1418.11
	s,kJ/(kg·K)	3.1939	3.9648	4.1974	4.4214	4.6390	4.8516	5.0598	5.2643	5.4653
0.20	v,m³/kg	0.2575	0.4089	0.4658	0.5222	0.5783	0.6342	0.6901	0.7458	0.8014
(197.9)	h,kJ/kg	593.49	761.74	836.62	917.58	1004.83	1098.49	1198.56	1304.98	1417.61
	s,kJ/(kg·K)	3.0902	3.7720	4.0061	4.2310	4.4492	4.6621	4.8706	5.0753	5.2765
0.40	v,m³/kg	0.1341	0.2012	0.2305	0.2592	0.2877	0.3160	0.3441	0.3721	0.4001
(214.3)	h,kJ/kg	609.25	757.62	833.52	915.14	1002.86	1096.86	1197.19	1303.80	1416.60
	s,kJ/(kg·K)	2.9896	3.5706	3.8079	4.0347	4.2541	4.4677	4.6768	4.8818	5.0833
0.70	v,m³/kg	0.07829	0.1122	0.1296	0.1465	0.1632	0.1796	0.1958	0.2120	0.2281
(229.8)	h,kJ/kg	622.15	751.24	828.77	911.44	999.88	1094.40	1195.12	1302.04	1415.07
	s,kJ/(kg·K)	2.9108	3.4008	3.6433	3.8730	4.0942	4.3090	4.5189	4.7245	4.9264
1.0	v,m³/kg	0.05502	0.07648	0.08926	0.1015	0.1133	0.1250	0.1365	0.1480	0.1593
(240.9)	h,kJ/kg	629.87	744.60	823.91	907.68	996.87	1091.93	1193.05	1300.27	1413.55
	s,kJ/(kg·K)	2.8603	3.2865	3.5345	3.7673	3.9904	4.2064	4.4171	4.6234	4.8257
2.0	v,m³/kg	0.02651	0.03451	0.04205	0.04882	0.05520	0.06135	0.06736	0.07326	0.07910
(265.8)	h,kJ/kg	640.20	720.03	806.77	894.73	986.63	1083.58	1186.08	1294.36	1408.47
	s,kJ/(kg·K)	2.7523	3.0353	3.3067	3.5512	3.7811	4.0015	4.2151	4.4233	4.6271
4.0	v,m³/kg	0.01051	0.01183	0.01813	0.02243	0.02614	0.02957	0.03283	0.03597	0.03904
(295.5)	h,kJ/kg	621.42	644.02	766.56	866.58	965.16	1066.43	1171.98	1282.52	1398.37
	s,kJ/(kg·K)	2.5659	2.6418	3.0271	3.3054	3.5520	3.7823	4.0022	4.2148	4.4218
7.0	v,m³/kg			0.00727	0.01106	0.01374	0.01604	0.01812	0.02009	0.02197
	h,kJ/kg			678.32	817.80	930.66	1039.90	1150.64	1264.88	1383.51
	s,kJ/(kg·K)			2.6637	3.0531	3.3357	3.5841	3.8149	4.0347	4.2466
10.	v,m³/kg			0.00397	0.00672	0.00892	0.01074	0.01235	0.01383	0.01523
	h,kJ/kg			589.90	764.34	894.94	1013.29	1129.67	1247.78	1369.25
	s,kJ/(kg·K)			2.3579	2.8446	3.1719	3.4411	3.6837	3.9109	4.1279
20.	v,m³/kg			0.00277	0.00342	0.00429	0.00521	0.00609	0.00691	0.00769
	h,kJ/kg			532.36	668.78	808.21	941.97	1071.47	1199.79	1329.20
	s,kJ/(kg·K)			2.0966	2.4756	2.8246	3.1289	3.3989	3.6458	3.8769
30.	v,m³/kg			0.00252	0.00287	0.00332	0.00384	0.00438	0.00492	0.00544
	h,kJ/kg			526.36	646.92	774.99	905.97	1037.43	1169.40	1302.75
	s,kJ/(kg·K)			2.0017	2.3367	2.6570	2.9549	3.2290	3.4829	3.7211



$H_1 = H_4 = 246$ kJ/kg

$H_3' = 726 \frac{kJ}{kg}$